Screening of the photon propagator in many-body optics

1970 J. Phys. A: Gen. Phys. 3 L24

(http://iopscience.iop.org/0022-3689/3/3/006)

mode was used on the outside of the plasma tube, and with a constant Δf value chosen (corresponding to a constant density value in the afterglow) the change in Q value was measured as the neutral pressure p in the tube was changed. A linear relationship between p and $1/Q$ resulted, thus indicating that the electron–neutral collision frequency was the measured quantity. Values of

$$\nu_e/p = 0{\cdot}70 \pm 0{\cdot}15 \times 10^9 \, \mathrm{s}^{-1} \, \mathrm{torr}^{-1}$$

for the hydrogen plasma and

$$\nu_e/p = 0{\cdot}80 \pm 0{\cdot}15 \times 10^9 \, \mathrm{s}^{-1} \, \mathrm{torr}^{-1}$$

for the helium plasma, were computed. The ion–neutral collision frequency ν_i proved to be a more difficult parameter to measure, and simple kinetic theory considerations on the hard-sphere model predict $\nu_i/\nu_e = v_i/4v_e$ where v_i and v_e are the ion and electron thermal velocities respectively. Thus, values of

$$\nu_i/p = 1{\cdot}0 \pm 0{\cdot}15 \times 10^6 \, \mathrm{s}^{-1} \, \mathrm{torr}^{-1}$$

for the helium plasma and

$$\nu_i/p = 1{\cdot}50 \pm 0{\cdot}15 \times 10^6 \, \mathrm{s}^{-1} \, \mathrm{torr}^{-1}$$

for the hydrogen plasma, were adopted.

Using these values, Re(ω) and the growth rate γ = Im(ω) were computed as a function of k_z from equation (1) for the particular conditions prevailing in each experiment. Typical theoretical curves are shown in figures 1(*a*) and 1(*b*). The full curves indicate the Re(ω) and refer to the left-hand scale, and the broken curves show the predicted growth rate γ and refer to the right-hand scale. It is seen that in this case good agreement is obtained, and similarly good agreement is obtained for other $m = +1$ and $m = +2$ results, in spite of the approximations in the theory and the possible errors in the experiment. Therefore, it is concluded that it is the drift-dissipative instability which is observed in these plasmas.

UKAEA Research Group,
Culham Laboratory,
Abingdon, Berks.

M. W. ALCOCK
B. E. KEEN
12th February 1970

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Screening of the photon propagator in many-body optics

Abstract. An internally consistent treatment of the interaction of light with a molecular fluid in terms of a screened photon propagator is reported. Screening simplifies the description of multiple scattering in terms of Ursell functions and the treatment of surface effects. In a translationally invariant approximate theory the screened photon propagator and the screened radiation reaction are expressed in terms of the refractive index.

Bullough (1968, 1969, Bullough *et al.* 1968, Bullough and Hynne 1968, to be referred to as I, II, III, IV respectively) has developed an internally consistent many-body theory for the optical properties of a molecular fluid in terms of the propagator for the free photon field

$$\mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega) \equiv (\nabla\nabla + k_0^2 \mathbf{U}) \frac{\exp(\mathrm{i}k_0 r)}{r}, \qquad r = |\boldsymbol{x}-\boldsymbol{x}'|, \qquad k_0 = \omega c^{-1} \tag{1}$$

where ω is the angular frequency and $\mathbf{U}$ is the unit tensor. The theory is a linear response theory. The propagator (1) emerges as a c-number commutator of a second quantized field theory and the radiation field is handled in detail. This theory, expressed in terms of (1), we shall refer to as the 'unscreened theory'.

Because light couples to a macroscopic system via the surface, the many-body system must be chosen finite (I, II). Surface effects therefore complicate all the intermolecular multiple scattering processes. Despite this complexity the macroscopic formulae for optical scattering cross sections obtained in III and IV are simple and (to a good approximation) agree with the phenomenological theory. Features of the microscopic theory suggest that it would exhibit a simpler structure in terms of a screened photon propagator. It is important, in any case, to understand the details of the optical screening processes.

Within the framework of Bullough (I, II, III, IV) we have therefore formulated an internally consistent theory in terms of a single screened photon propagator, $\mathscr{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$. This 'screened' theory is wholly consistent with the unscreened theory and describes the same system. In particular the system must be finite and surface effects remain, partly concealed in $\mathscr{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$. However, we also describe a natural and very good approximation to a translationally invariant theory, in which $\mathscr{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$ has the natural closed form

$$\tilde{\mathbf{F}}(\boldsymbol{x}, \boldsymbol{x}'; \omega) \equiv m^{-2}(\omega)\{\nabla\nabla + m^2(\omega) k_0^2 \mathbf{U}\} \frac{\exp\{\mathrm{i}m(\omega)k_0 r\}}{r} \tag{2}$$

where $m(\omega)$ is the refractive index of the unscreened theory (III). The propagator $\tilde{\mathbf{F}}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$ has been used before by Fixman (1955), Mazur (1958) and Bullough (1965, 1967, 1969). Both our integral equation and its translationally invariant solution (2) are fundamental to the bulk binding energy theory of Dzyaloshinskii *et al.* (Abrikosov *et al.* 1965).

The contribution of the screened theory we report here is therefore to show how the screening processes can be said to occur, how these are related to surface effects, how they can be handled consistently in a (complex) refractive index theory, and how the propagator (2) and a translationally invariant approximation can be extracted from the theory.

We now outline the theory. We consider an 'instantaneous' situation in which a system of molecules at fixed sites $\boldsymbol{x}_j$, enclosed in the finite region V, is subjected to an external electric field $\boldsymbol{E}(\boldsymbol{x}, \omega)$. In III the argument from the quantized field theory admits a natural definition of the induced dipole moments, $\boldsymbol{P}_j$. Then, in terms of the 'instantaneous density'

$$\rho(\boldsymbol{x}) = \sum_j \delta(\boldsymbol{x}-\boldsymbol{x}_j)$$

and the instantaneous polarization

$$\boldsymbol{P}^{\mathrm{in}}(\boldsymbol{x}, \omega) = \sum_j \boldsymbol{P}_j \delta(\boldsymbol{x}-\boldsymbol{x}_j)$$

the result of III is

$$\boldsymbol{P}^{\text{in}}(\boldsymbol{x}, \omega) = \rho(\boldsymbol{x})\alpha(\omega)\{\boldsymbol{E}(\boldsymbol{x}, \omega) + \int_V \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega) \,.\, \boldsymbol{P}^{\text{in}}(\boldsymbol{x}', \omega)\, \mathrm{d}\boldsymbol{x}'\}. \tag{3}$$

The isotropic polarizability of the isolated particles, $\alpha(\omega)$, is real and the radiation reaction is conveniently incorporated as a self-correlation by the 'convergent part integral' definition (III)

$$\int \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)\delta(\boldsymbol{x}-\boldsymbol{x}')\, \mathrm{d}\boldsymbol{x}' \equiv \tfrac{2}{3}\mathrm{i}k_0{}^3\,\mathbf{U}. \tag{4}$$

Equation (3) is the fundamental equation of classical molecular optics (Rosenfeld 1951) in a convenient form and from this point our discussion is essentially classical. We are interested in the ensemble average $\boldsymbol{P}(\boldsymbol{x}, \omega) \equiv \langle \boldsymbol{P}^{\text{in}}(\boldsymbol{x}, \omega)\rangle_{\text{av}}$, and introduce the average electric field by the natural definition (Mazur 1958, Bullough 1970 to be published)

$$\mathscr{E}(\boldsymbol{x}, \omega) \equiv \boldsymbol{E}(\boldsymbol{x}, \omega) + \int_V \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega) \,.\, \boldsymbol{P}(\boldsymbol{x}', \omega)\, \mathrm{d}\boldsymbol{x}'. \tag{5}$$

The integral over V is initially undefined. We define it as the sum of the conditionally convergent integral obtained when a vanishingly small spherical region v around $\boldsymbol{x}$ is excluded from the integration, and the contribution from v, defined by means of the generalized function interpretation (III)

$$\lim_{v\to 0}\int_v \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)\, \mathrm{d}\boldsymbol{x}' \equiv -\frac{4\pi}{3}\,\mathbf{U}. \tag{6}$$

Equations (3) and (5) enable us to eliminate the *external* field, and we obtain

$$\boldsymbol{P}^{\text{in}}(\boldsymbol{x}, \omega) = \rho(\boldsymbol{x})\alpha(\omega)[\mathscr{E}(\boldsymbol{x}, \omega) + \int_V \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega).\{\boldsymbol{P}^{\text{in}}(\boldsymbol{x}', \omega) - \langle \boldsymbol{P}^{\text{in}}(\boldsymbol{x}', \omega)\rangle_{\text{av}}\}\, \mathrm{d}\boldsymbol{x}']. \tag{7}$$

This is the fundamental integral equation of the present theory.

Iteration of (7) shows that we can write

$$\boldsymbol{P}^{\text{in}}(\boldsymbol{x}, \omega) = \int_V \mathbf{\Lambda}^{\text{in}}(\boldsymbol{x}, \boldsymbol{x}'; \omega).\mathscr{E}(\boldsymbol{x}', \omega)\, \mathrm{d}\boldsymbol{x}'. \tag{8}$$

The statistical average $\mathbf{\Lambda}(\boldsymbol{x}, \boldsymbol{x}'; \omega) \equiv \langle \mathbf{\Lambda}^{\text{in}}(\boldsymbol{x}, \boldsymbol{x}'; \omega)\rangle_{\text{av}}$ is found to all orders in the small parameter $n\alpha(\omega)$ where $n \equiv \langle \rho(\boldsymbol{x})\rangle_{\text{av}}$

$$\begin{aligned}\mathbf{\Lambda}(\boldsymbol{x}_1, \boldsymbol{x}_0; \omega) = {} & n\alpha(\omega)\,\mathbf{U}\delta(\boldsymbol{x}_1 - \boldsymbol{x}_0) + n^2\alpha^2(\omega)\mathbf{F}_{10}H_{10} \\ & + \sum_{p=3}^{\infty} n^p\alpha^p(\omega)\int_V \cdots \int_V \mathbf{F}_{12}\,.\,\mathbf{F}_{23}\,\ldots\,.\,\mathbf{F}_{p-10}H_{123\ldots p-10}\,\mathrm{d}\boldsymbol{x}_2\ldots\mathrm{d}\boldsymbol{x}_{p-1}\end{aligned}$$

$$H_{(p)} = n^{-p}\sum_{t=1}^{p}(-1)^{t-1}\sum_{\text{lin}(t)}\prod_{s=1}^{t}\mathscr{G}_{(q_s)}, \qquad \sum_{s=1}^{t} q_s = p. \tag{9}$$

The sum over $\text{lin}(t)$ is taken over all 'linear' partitions of the ordered set (p) of p indices into t subsets (q_s) of q_s *consecutive* elements from (p).

$$\mathscr{G}_{ijk\ldots l} \equiv \langle \rho(\boldsymbol{x}_i)\rho(\boldsymbol{x}_j)\rho(\boldsymbol{x}_k)\ldots\rho(\boldsymbol{x}_l)\rangle_{\text{av}}$$

is the generalized correlation function which includes all self-correlations, the sources of the radiation damping terms of the series. To save space we use indices to denote position variables. This solution (9) cannot simply be extended to an infinite system since the integrals then diverge. It is related to the refractive index: we find we can identify the integrated power series for $\mathbf{\Lambda}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$ with the series for $m^2(\omega)-1$ in the microscopic refractive index theory (III), with the result

$$4\pi \boldsymbol{uu}: \int_V \mathbf{\Lambda}(\boldsymbol{x}, \boldsymbol{x}'; \omega) \exp\{im(\omega)\boldsymbol{k}_0.(\boldsymbol{x}'-\boldsymbol{x})\} \,\mathrm{d}\boldsymbol{x}' = m^2(\omega)-1 \tag{10}$$

where V is a parallel-sided slab with sides normal to $\boldsymbol{k}_0$ (the wave vector of the incident light). The unit vector $\boldsymbol{u}$ is a polarization direction and $\boldsymbol{u}\,.\,\boldsymbol{k}_0 = 0$. The result (10) justifies our choice (5). Actually, the left-hand side of (10) is slightly $\boldsymbol{x}$-dependent when $\boldsymbol{x}$ is close to the surface, but virtually $\boldsymbol{x}$-independent otherwise as described in III. There it was necessary to analyse the surface-dependent terms piece by piece to find their position-independent contributions to the refractive index. However, by introducing a screened photon propagator $\mathscr{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$ we can formally sum these surface terms. The propagator is defined as the Neumann solution of the integral equation (Abrikosov *et al.* 1965, Bullough 1970 to be published).

$$\mathscr{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega) = \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega) + \int_V \int_V \mathbf{F}(\boldsymbol{x}, \boldsymbol{x}''; \omega).\mathbf{\Lambda}(\boldsymbol{x}'', \boldsymbol{x}'''; \omega).\mathscr{F}(\boldsymbol{x}''', \boldsymbol{x}'; \omega)\,\mathrm{d}\boldsymbol{x}''\,\mathrm{d}\boldsymbol{x}''' \tag{11}$$

with the definition (6). We use the series solution (9) for $\mathbf{\Lambda}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$.

With the closed form (11) we get a non-linear integral equation for $\mathbf{\Lambda}^{\mathrm{in}}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$. We have found the averaged solution of this equation to be

$$\begin{aligned}\mathbf{\Lambda}(\boldsymbol{x}_1, \boldsymbol{x}_0; \omega) = {} & n\alpha(\omega)\,\mathbf{U}\delta(\boldsymbol{x}_1-\boldsymbol{x}_0) + n^2\alpha^2(\omega)\mathscr{F}_{10}Y_{10} \\ & + \sum_{p=3}^{\infty} n^p\alpha^p(\omega) \int_V \dots \int_V \mathscr{F}_{12}.\mathscr{F}_{23} \dots .\mathscr{F}_{p-10}Y_{123\dots p-10} \\ & \times \mathrm{d}\boldsymbol{x}_2 \dots \mathrm{d}\boldsymbol{x}_{p-1}.\end{aligned} \tag{12}$$

Here $Y_{(p)} = n^{-p}\mathscr{Y}_{(p)}$, and $\mathscr{Y}_{(p)}$ is given by the generalized Ursell functions $\mathscr{U}_{(q)}$ (see III), which include all self-correlations, as

$$\mathscr{Y}_{(p)} = \sum_{\mathrm{con}} \prod_{s} \mathscr{U}_{(q_s)}, \qquad \sum_{s} q_s = p. \tag{13}$$

The sum in (13) is taken over all 'connected partitions' of the ordered set (p) of p indices into subsets (q_s) of q_s elements. We define a 'connected partition': we represent an ordered set (p) by consecutive vertices of a regular polygon, and a partition of (p) by the collection of polygons formed by the lines connecting in cyclic order the vertices representing the indices of each set. Then a partition of (p) is *connected* if, and only if, any line drawn through the graph crosses a line of a set-polygon. The first few $\mathscr{Y}_{(p)}$ functions are:

$$\mathscr{Y}_{12} = \mathscr{U}_{12}, \qquad \mathscr{Y}_{123} = \mathscr{U}_{123}, \qquad \mathscr{Y}_{1234} = \mathscr{U}_{1234} + \mathscr{U}_{13}\mathscr{U}_{24}.$$

We use a version of the diagram notation of III for a clear exposition of the result (12). A circle represents a factor $n\alpha(\omega)$ and a position variable. A line represents $\mathbf{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$ and a double line represents $\mathscr{F}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$. A loop of dotted lines (indicated by a cross line when coinciding with full lines) represents an Ursell function. All position variables are integrated except the first one and except those represented by circles carrying a dot. In this representation $\mathbf{\Lambda}(x_1, x_0; \omega) - n\alpha(\omega)\mathbf{U}\delta(x_1 - x_0)$ is given by figure 1(*a*).

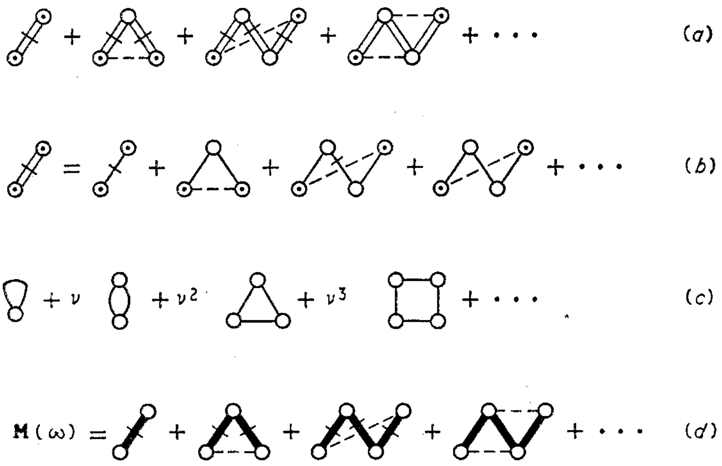

Figure 1. Diagram representation of multiple scattering terms.

When the Neumann solution is substituted for $\mathscr{F}(x, x'; \omega)$ we do indeed recover the solution (9) for $\mathbf{\Lambda}(x, x'; \omega)$. In figure 1(*b*) we quote the expansion of the screened two-body term up to fourth order to exemplify how each term in figure 1(*a*) conceals an infinite series of 'surface terms' of the unscreened theory. We next show how these series can be approximately summed. When, in equation (11), we replace $\mathbf{\Lambda}(x, x'; \omega)$ by $\{m^2(\omega) - 1\}(4\pi)^{-1}\mathbf{U}\delta(x - x')$ (which is 'local' and satisfies (10)) and V by 'all space' we obtain an equation, which has the solution $\tilde{\mathbf{F}}(x, x'; \omega)$, given by (2). Motivated by the intuitively acceptable form of $\tilde{\mathbf{F}}(x, x'; \omega)$ we now define the bulk approximation as the description in which $\tilde{\mathbf{F}}(x, x'; \omega)$ replaces $\mathscr{F}(x, x'; \omega)$, and where all integrations are taken over all space. We need expressions for $\tilde{\mathbf{F}}(x, x'; \omega)$ corresponding to (4) and (6) for $\mathbf{F}(x, x'; \omega)$. We find

$$\int \tilde{\mathbf{F}}(x, x'; \omega)\delta(x - x')\,\mathrm{d}x' = \tfrac{2}{3}\mathrm{i}m(\omega)k_0^3\mathbf{U} \tag{14}$$

$$\lim_{v \to 0} \int_v \tilde{\mathbf{F}}(x, x'; \omega)\,\mathrm{d}x' = -m^{-2}(\omega)\frac{4\pi}{3}\mathbf{U}. \tag{15}$$

Figure 1(*c*) shows the series that leads to the expression (14) for the screened radiation reaction in the bulk approximation. In (*c*), $\nu \equiv \{m^2(\omega) - 1\}/4\pi n\alpha(\omega)$ and the first term represents the radiation reaction term (4). The screened radiation reaction (14) is of fundamental interest: it generalizes the one-particle self-energy, based on (4), to the many-particle system (I, Bullough 1970 to be published). We can now obtain an equation for the refractive index from (12) and (10):

$$m^2(\omega) - 1 = 4\pi \boldsymbol{uu}\colon \{n\alpha(\omega)\mathbf{U} + \mathbf{M}(\omega)\} \tag{16}$$

where the tensor $\mathbf{M}(\omega)$ is given in figure 1(d). This result compares with equation (11) of III. As in III each term contains the exponential factor which comes from (10); but now a heavy line represents $\tilde{\mathbf{F}}(\boldsymbol{x}, \boldsymbol{x}'; \omega)$ and the integrations are taken over all space. The combination of terms at order p is determined by equation (13). Note particularly that (16) contains the $\mathscr{Y}$ functions and not the Ursell functions simply. Therefore the division, in III, of terms into surface-independent terms controlled by Ursell functions and 'surface terms' does not sufficiently categorize the terms: 'surface terms' remain at fourth and higher orders in (16). But these all converge because of the *combination* of Ursell functions and photon propagators. Thus the screening process is entirely due to all of the surface-dependent 'surface terms', which do not have this property. Then (16) is part of a translationally invariant theory with $\boldsymbol{k}_0$ and $\boldsymbol{u}$ orthogonal but otherwise with arbitrary directions.

This analysis also enables us to report that the extended Einstein equation of IV is certainly valid up to $O(n^4\alpha^4(\omega))$ and removes the qualification beyond $O(n^3\alpha^3(\omega))$ noted in IV.

The details of this work will appear in the series of papers entitled 'Many-body optics' (see already I, II, Bullough 1970 to be published).

We are very indebted to Dr. R. K. Bullough for suggesting the problem of a consistent screened theory and for many helpful discussions.

Chemistry Laboratory, III,
H. C. Ørsted Institute,
Copenhagen,
Denmark.

F. HYNNE
4th February 1970

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Experimental evidence of a minimum in the thermal conductivity against composition curve for the He–HD mixture

Abstract. The thermal conductivity against composition curve for a He–HD mixture at 297 K shows a well-pronounced minimum giving a λ value for the equimolar mixture lower by about 4% than that of both the pure components. Although such behaviour is quite exceptional there is no reason to consider it as anomalous.